## Understanding Profit Max

Econ 201/Haworth

In the Profit Max process for a price setter, the goal is to produce an output level that maximizes the firm's profit, and then set a price which allows the firm to sell that specific number of units.

Therefore, the generic Profit Max process for a price setting firm, not necessarily a monopoly is:

1. Firm chooses output
2. Firm sets price
3. Firm calculates profit (of course, if the firm earns a loss, it'll also check "shut down")

How much should the firm produce? Let $\mathrm{q}=$ output of a firm
Recall from the Perfect Competition section that the $q$ chosen by a firm will be an equilibrium value (i.e. once we find $q^{*}$, the firm earns max profit and there's no reason to change $q$ ).

If it's an equilibrium, then we can use $\mathrm{MB}=\mathrm{MC}$. I.e., we can find $\mathrm{q}^{*}$ by finding where the Marginal Benefit (MB) of producing a certain level of output is equal to the Marginal Cost (MC) of producing that level of output.

Question is - what is benefit and cost here?

- Benefit $=$ Revenue from selling units (i.e. $P \cdot q$, where $P=$ price and $q=$ output)
- Cost $=$ Cost of selling units (i.e. Total Cost, or TC)

That implies the following about MB and MC

- Marginal Benefit $(\mathrm{MB})=>$ Marginal Revenue, which is $M R=\frac{\Delta T R}{\Delta q}$
- $\quad$ Marginal Cost $(\mathrm{MC}) \Rightarrow M C=\frac{\Delta T C}{\Delta q}$

Therefore, a general way of finding $q^{*}$, the output that maximizes a firm's profit is to use the equilibrium condition $M R=M C$, where $M C$ is the same $M C$ we discussed throughout perfect competition, and MR is a new term.

Note that in Perf Comp, our equilibrium condition was $\mathrm{P}=\mathrm{MC}$

That leads to a question: how can we resolve the fact that $\mathrm{P}=\mathrm{MC}$ is the profit max condition for perfect competition and $\mathrm{MR}=\mathrm{MC}$ is the profit max condition for all firms, not just Perf Comp?

What we will do next involves some math, but what's important is the punch line.

Let's note first that whether it's $\mathrm{MR}=\mathrm{MC}$ or $\mathrm{P}=\mathrm{MC}$, we have MC on the right hand side of both equations, so we really need to focus on trying to explain MR.

Start with Total Revenue (TR). Remember that we calculate total revenue as follows:
$T R=P \cdot q$

If there's a change in $T R$, then we can express that as follows: $\Delta T R=(\Delta P \cdot q)+(\Delta q \cdot P)$ (you'll just have to trust me on that last statement, I'm not going to prove it)

Suppose we divide both sides of $\Delta T R=(\Delta P \cdot q)+(\Delta q \cdot P)$ by $\Delta q$ ? That would give us:
$\frac{\Delta T R}{\Delta q}=\left(\frac{\Delta P}{\Delta q} \cdot q\right)+\left(\frac{\Delta q}{\Delta q} \cdot P\right)$

Let's note a couple things.
(1) $M R=\frac{\Delta T R}{\Delta q}$ and so we can substitute MR for what's on the left side of the $=$
(2) $\frac{\Delta q}{\Delta q}=1$ and so we can replace $\left(\frac{\Delta q}{\Delta q} \cdot P\right)$ with $(P)$

That gives us this equation:
$M R=\left(\frac{\Delta P}{\Delta q} \cdot q\right)+P$

What does $\frac{\Delta P}{\Delta q}$ represent?
(remember, that expression actually a derivative, as in ... it involves calculus, so yay)
$\frac{\Delta P}{\Delta q}$ tells us how the price (market price if it's a perfectly competitive firm, firm's price if it's a price setting firm) changes whenever the firm produces a different amount of output.

Go back to what we said in the perfect competition section. Perfectly competitive firms are price takers, and price takers are firms which are so small, they won't affect the market price by changing their output.

What that means is this. For a perfectly competitive (price taking) firm, $\frac{\Delta P}{\Delta q}=0$
Therefore, substituting 0 for $\frac{\Delta P}{\Delta q}$ into our equation for MR , we have $\mathrm{MR}=\mathrm{P}$
Whether you followed the math or not, here's what that means.

If $\mathrm{MR}=\mathrm{P}$ whenever we're dealing with a perfectly competitive firm, then we can express that firm's profit maximizing equation as saying the firm can produce where $\mathrm{P}=\mathrm{MC}$, or that the firm can produce where $\mathrm{MR}=\mathrm{MC}$. I.e. both equalities say the exact same thing since $\mathrm{MR}=\mathrm{P}$ in Perfect Competition.

What about price setting firms like Monopoly?
Go back to our MR equation from above:
$M R=\left(\frac{\Delta P}{\Delta q} \cdot q\right)+P$

As we said in the "characteristics of Monopoly" section, monopoly firms are price setters, not price takers. That suggests $\frac{\Delta P}{\Delta q} \neq 0$. Ok, but what would $\frac{\Delta P}{\Delta q}$ be equal to with a price setter?

We can answer that question intuitively. If a price setting firm wants to sell more units (i.e. increase q), then how would the firm do that? They'd obviously have to follow the Law of Demand and lower the price. In other words, we know that in order to sell more units, a price setting firm would have to lower price. If the firm wanted to sell fewer units, the firm would raise price. All of that implies $\frac{\Delta P}{\Delta q}<0$.

This tells us 2 things.
(1) All firms have a demand curve, just like markets. The firm's demand curve tells the firm how changing price affects the number of units the firm will sell. If we recall that the slope of any demand curve is $\frac{\Delta P}{\Delta q}$ (i.e. rise/run), then we know that $\frac{\Delta P}{\Delta q}$ tells us the slope of any respective firm's demand curve. If it's a perfectly competitive firm, then $\frac{\Delta P}{\Delta q}=0$ and the slope of the perfectly competitive firm's demand curve is 0 . In other words, it's a horizontal line. If it's a price setting firm like monopoly, then $\frac{\Delta P}{\Delta q}<0$ and the slope of the price setting firm's demand curve is negative.

Here is all of that represented on a pair of graphs.

q
Perfectly Competitive (or Price taking) firm's demand curve

q
Price setting firm's demand curve
(2) In our goal to better understand MR, we discover that with perfectly competitive firms, $\mathrm{MR}=\mathrm{P}$, which means that if we draw a marginal revenue curve on the graph for a perfectly competitive firm, then the MR curve is a horizontal line on that graph, just like the firm's demand curve. They would both be a horizontal line at $\mathrm{P}^{*}$. When we have a price setting firm, then we know $\mathrm{MR}<\mathrm{P}$, which means that the MR curve lies below the firm's demand curve. Both are negatively sloped, but the MR curve would be steeper.

Here is all of that represented on a pair of graphs.

q
Perfectly Competitive firm's demand \& MR curves


